2015 Junior Kangaroo Solutions

1. C Ben, his father, his mother, his sister, his brother and all four of the parrots have two legs each, making 18 legs in total. The two dogs and the three cats have four legs each, making 20 legs in total. Hence there are 38 legs in the house.

2. A Let the five consecutive integers be $n - 2$, $n - 1$, n, $n + 1$ and $n + 2$. These have a sum of $5n$. Hence $5n = 2015$ and therefore $n = 403$. Therefore, the smallest integer is $403 - 2 = 401$.

3. E From the diagram, the ant walks the equivalent of five edges. Therefore the ant walks $5 \times 18\text{ cm} = 90\,\text{cm}$.

4. C One day contains 24 hours. Hence $\frac{1}{8}$ of a day is three hours and $\frac{1}{6}$ of this is half an hour. Half an hour contains $\left(\frac{1}{2} \times 60 \times 60\right)$ seconds $= 1800$ seconds and $\frac{1}{4}$ of this is 450 seconds. Hence there are 450 seconds in the required fraction of a day.

5. B Long division gives

$$\begin{array}{r|l} & 101 \\ \hline 2015 & 203515 \\ & 2015 \\ \hline & 2015 \\ & 2015 \end{array}$$

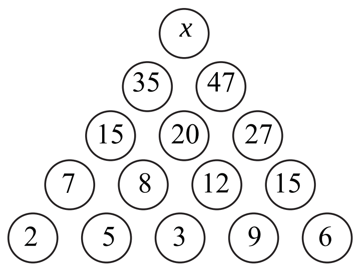
Hence $203\,515 \div 2015 = 101$.

6. C Look first at the numbers labelling the left- and right-hand sides of the rectangles. It can be seen that only rectangles *A*, *C* and *E* can be arranged in a row of three with their touching sides equal and so they must form the top row of the diagram. The only common value on the right- and left-hand sides of rectangles *B* and *D* is 3 and so rectangle *D* will be placed in position IV. Therefore, the rectangle to be placed in position I needs to have 2 on its lower edge. Hence rectangle *C* should be placed in position I (with A in position II, E in position III and B in position V).

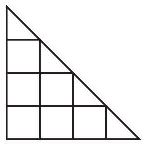
7. C Each time Selina cuts up a piece of paper, she turns one piece into ten smaller pieces and so the number of pieces she has increases by nine. Selina makes four cuts so the total number of pieces she finishes with is $1 + 4 \times 9 = 37$.

8. A It takes John 24 minutes to run both ways so it will take him $\frac{1}{2} \times 24$ minutes $= 12$ minutes to run one way. Also, it takes him 40 minutes to walk one way and run the other so walking one way takes him $(40 - 12)$ minutes $= 28$ minutes. Hence it will take 2×28 minutes $= 56$ minutes to walk both ways.

9. B The empty spaces in the diagram can be completed as shown. Hence the value of x is $35 + 47 = 82$.



10. **D** In the diagram, there are six 1×1 squares and one 2×2 square. There are also four triangles that are half of a 1×1 square, three triangles that are half of a 2×2 square, two triangles that are half of a 3×3 square and one triangle that is half of a 4×4 square. Hence $S = 7$ and $T = 10$ so $S \times T = 70$.



11. **B** Let b cm be the length of the base and let h cm be the height of the small equilateral triangles. The area of each triangle is 4 cm^2 so $\frac{1}{2} \times b \times h = 4$.

The shaded area is a trapezium with parallel sides of length $4h$ and $5h$ and with distance $\frac{5}{2}b$ between the parallel lines. Using the formula for the area of a trapezium, the shaded area is equal to $\frac{1}{2}(4h + 5h) \times \frac{5}{2}b = \frac{1}{4} \times 45bh$. From above, $bh = 8$ so this area is equal to $\frac{1}{4} \times 45 \times 8$ cm^2 $= 90$ cm^2.

(Alternatively, one could observe that the first four horizontal rows of the shaded region have area equivalent to five of the small equilateral triangles while the fifth layer has area equivalent to half of that or 2.5 equilateral triangles. Hence the shaded region has area equivalent to 22.5 small equilateral triangles and so has area 22.5×4 cm^2 $= 90$ cm^2.)

12. **C** The maximum value one can obtain by adding a two-digit number and two one-digit numbers is $99 + 9 + 9 = 117$. Hence the triangle must represent 1 and therefore the sum of the three numbers is 111. To obtain this answer to the sum, the circle must represent 9 since $89 + 9 + 9$ is less than 111. Hence the two squares must sum to $111 - 99 = 12$. Therefore the square represents the digit 6.

13. **A** To have one of the integers in the sum as large as possible, the other nine must be as small as possible. The minimum possible sum of nine distinct positive integers is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$. Hence the largest possible integer in the sum is $100 - 45 = 55$.

14. **D** The shaded area is equal to that of 1 circle + $4 \times \frac{1}{4}$ circles = 2 circles. The area of the unshaded parts of the circles is equal to that of $4 \times \frac{3}{4}$ circles = 3 circles. Hence the required ratio is 2 : 3.

15. **C** Let the length and width of the garden be a metres and b metres respectively and let the width of the path be x metres. The perimeter of the garden is $2(a + b)$ metres and the perimeter of the larger rectangle formed by the garden and the path is $2(a + 2x + b + 2x)$ metres. Hence the difference between the distance along the outside edge of the path and the perimeter of the garden in metres is $2(a + 2x + b + 2x) - 2(a + b) = 8x$. Therefore $8x = 24$ which has solution $x = 3$. Hence the width of the path is 3 metres.

16. **C** The caterpillar will be as far away as possible from its hole if, at each turn, it always heads away from the hole. Hence its maximum distance will occur when it has travelled $(2 + 4 + 6 + 8)$ metres = 20 metres in one direction and $(3 + 5 + 7)$ metres = 15 metres in a perpendicular direction. Using Pythagoras' Theorem, the maximum distance in metres is then $\sqrt{20^2 + 15^2} = \sqrt{625} = 25$.

17. **B** To obtain the smallest number of gold coins, the least possible number of boxes must be opened. Therefore, Blind Pew must open the trunk and all five chests, leaving only three boxes to be opened. Hence the smallest number of gold coins he could take is $3 \times 10 = 30$.

18. **E** Let the integer Brian chooses be x. Following the operations in the question, his final result is $2(4x - 30) - 10$. His answer is a two-digit number so $9 < 2(4x - 30) - 10 < 100$. Hence we have $9 < 8x - 70 < 100$ which has solution $9\frac{7}{8} < x < 21\frac{1}{4}$. Therefore the largest integer Brian could choose is 21.

19. D Consider the times when the poster does not tell the truth. The poster will not tell the truth one hour before Clever Cat changes his activity and will remain untrue until he has been doing his new activity for two hours. Hence the poster does not tell the truth for three hours around each change of activity but tells the truth the rest of the time. Therefore, the poster will tell the truth for $(24 - 2 \times 3)$ hours $=$ 18 hours.

20. B The nth term of the sequence 1, 3, 5, … is $2n - 1$. Therefore at the start of the solution the total number of tiles in the 15th shape is $(2 \times 15 - 1)^2 = 29^2 = 841$. In each shape, there is one more black tile than white tile. Hence there would be $\frac{1}{2}(841 + 1) = 421$ black tiles in the 15th shape.

21. E The digits in the code are all different, so the result of dividing the second digit by the third cannot be 1. The first digit is a square number and, since it cannot be 1, is 4 or 9. The largest possible code will start with 9 and have second digit ÷ third digit = 3 and is 962. The smallest possible code will start with 4 and have second digit ÷ third digit = 2 and is 421. Hence the difference between the largest and smallest possible codes is $962 - 421 = 541$.

22. D Each cube has volume $3^3\ \text{cm}^3 = 27\ \text{cm}^3$. There are four cubes visible in the base layer from both the front and the side so the maximum number of cubes in the base layer is $4 \times 4 = 16$. Similarly, the maximum number of cubes in the second layer is $2 \times 2 = 4$. Hence the maximum number of cubes in the solid is $16 + 4 = 20$ with a corresponding maximum volume of $20 \times 27\ \text{cm}^3 = 540\ \text{cm}^3$.

23. E It can be shown that a positive integer has an odd number of factors if and only if it is square. The smallest three-digit square number is $10^2 = 100$ and the largest is $31^2 = 961$. Hence there are $31 - 9 = 22$ three-digit numbers which have an odd number of factors.

24. E The question tells us that Sally is not sitting at either end. This leaves three possible positions for Sally, which we will call positions 2, 3 and 4 from the left-hand end. Were Sally to sit in place 2, neither Dolly nor Kelly could sit in places 1 or 3 as they cannot sit next to Sally and, since Elly must sit to the right of Dolly, there would be three people to fit into places 4 and 5 which is impossible. Similarly, were Sally to sit in place 3, Dolly could not sit in place 2 or 4 and the question also tells us she cannot sit in place 1 so Dolly would have to sit in place 5 making it impossible for Elly to sit to the right of Dolly. However, were Sally to sit in place 4, Dolly could sit in place 2, Kelly in place 1, Molly (who cannot sit in place 5) in place 3 leaving Elly to sit in place 5 at the right-hand end.

25. A Carol finishes 25 metres behind Bridgit, so she travels 75 metres while Bridgit runs 100 metres. Therefore she runs 3 metres for every 4 metres Bridgit runs. When Anna finishes, Bridgit has run 84 metres, so that at that time Carol has run $\frac{3}{4} \times 84$ metres $=$ 63 metres. Hence Carol finishes $(100 - 63)$ metres $=$ 37 metres behind Anna.